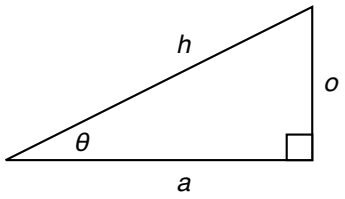
Note 1a Trigonometry

Trigonometry is the mathematics of the triangle. The simplest triangle is the right triangle.

Trigonometric Functions

Given a right triangle, you can relate an angle "θ" to the sides in the following way.



The longest side is the hypotenuse "h". The side next to the angle of interest is the adjacent side "a". The side facing the angle is the opposite side "o". They are related this way.

$$\frac{o}{h} = \sin\theta \quad and \quad \frac{a}{h} = \cos\theta \quad and \quad \frac{o}{a} = \tan\theta$$

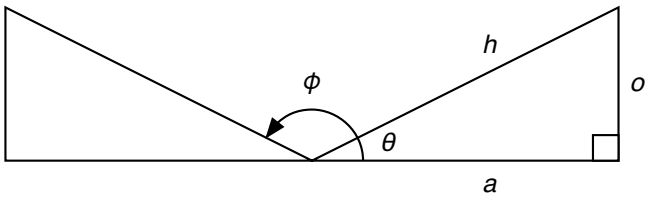
Inverse-Trigonometric Function

The inverses of these function are

$$\theta = \sin^{-1}\left(\frac{o}{h}\right) \quad and \quad \theta = \cos^{-1}\left(\frac{a}{h}\right) \quad and \quad \theta = \tan^{-1}\left(\frac{o}{a}\right)$$

Each of these functions has two solutions. For the sine function,

$$\theta \ and \ \phi = \sin^{-1}\left(\frac{o}{h}\right)$$

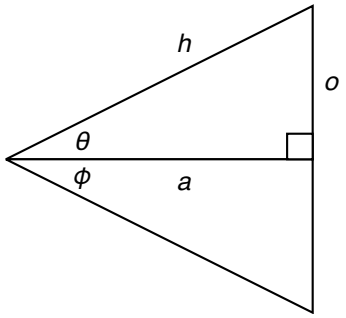


And the two angles are related this way.

$$\phi = \pi - \theta$$

For the cosine function,

$$\theta \ and \ \phi = \cos^{-1}\left(\frac{o}{h}\right)$$

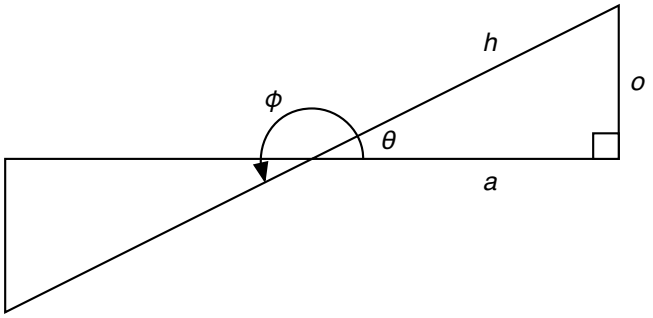


And the two angles are related this way.

$$\phi = -\theta$$

For the tangent function,

$$\theta \ and \ \phi = \tan^{-1}\left(\frac{o}{h}\right)$$

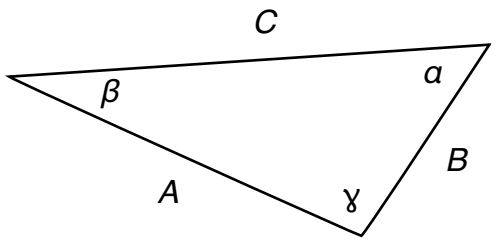


And the two angles are

$$\phi = \pi + \theta$$

Trigonometric Identities

For general triangles, there are two sets of relationships between the angles and the sides.



The first set of relationships is called the law of sine.

$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$

The second set of relationships is called the law of cosine. The sides and angles can be rotated to the other versions.

$$A^2 = B^2 + C^2 - 2|B||C|\cos\alpha$$

$$B^2 = C^2 + A^2 - 2|C||A|\cos\beta$$

$$C^2 = A^2 + B^2 - 2|A||B|\cos\gamma$$