

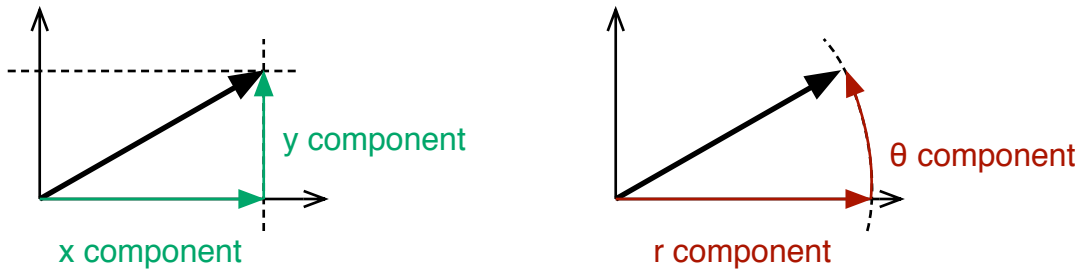
Note 1b Vectors

Vectors are representations of physical measurements that have **spacial** directionality. These include quantities such as position, velocity, acceleration, and force. A vector is represented by an arrow pointing in the direction that the measurement is pointing. The length of the arrow indicates the size, or the magnitude, of the measurement. The orientation of the arrow indicates the angle or the direction of the measurement.

Coordinate Representations

In order to define vectors, you must first define a coordinate system.

We will use the following two types of coordinate systems to describe vectors. They are the rectilinear or the cartesian coordinates (x-y) and the polar (r- θ) coordinates in two dimensions.



The way to express the above vector in the rectilinear coordinates is this.

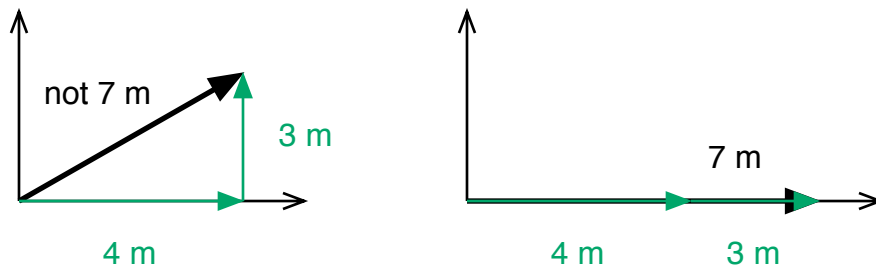
$$(4\text{ m}) \cdot \hat{i} + (3\text{ m}) \cdot \hat{j}$$

Or this shorter version.

$$\langle 4\text{ m}, 3\text{ m} \rangle$$

The two parts being added together are called **components**. The symbols after the distances are called **unit vectors**. They indicate only the direction. The way to read the vector above is to say “4 meters in the x direction and 3 meters in the y direction”.

The two components can not be combined because they point in different directions. Your distance from the origin is not 7 m meters if you walk 4 meter in the x direction then 3 meter in the y direction. Components in the same direction do add.



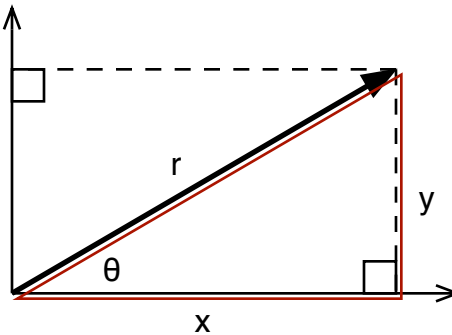
In polar coordinates, the above vector is written like this.

$$(5 \text{ m}) @ (36.87^\circ) \text{ or } (5 \text{ m}) @ (0.6435 \text{ rad})$$

The first component here is the radial component or the **magnitude**. The second component is the **angle** or the tangential component. The way to read this is to say “5 meters from the origin at an angle of 36.87 degrees (or 0.6435 radian)”. The angle, by convention, is measured counter-clockwise from the positive x axis.

Converting Between Representations

Since the two representations are equivalent, you can take a vector in one representation and change it to the other. The key is the right triangle made from the perpendicular projections (dotted lines) from the end of the vector to the coordinate axes.



Given the x and y coordinates, we can convert to the polar coordinates using the following transformations.

$$r = \sqrt{x^2 + y^2} \text{ and } \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Note that the inverse trigonometric function have two possible results, but the calculator will return only one of them. You have to know which quadrant you are in in order to select the correct answer.

$$\theta = \tan^{-1}\left(\frac{3 \text{ m}}{4 \text{ m}}\right) = 36.87^\circ \text{ (quadrant I)}$$

$$\theta = \tan^{-1}\left(\frac{-3 \text{ m}}{-4 \text{ m}}\right) = 216.87^\circ \text{ (quadrant III)}$$

Given the r and θ coordinates, we can convert to the rectilinear coordinates using the following transformations.

$$x = r \cos(\theta) \text{ and } y = r \sin(\theta)$$

Vector Arithmetic

There are two rules you have to remember when you are adding (or subtracting) vectors.

1. Only Cartesian components in the same direction can be combined into a single value.
2. This means that you can add vectors algebraically only in Cartesian coordinates.

For example, let there be two displacements. I present them here in polar coordinates.

$$\Delta \vec{s}_1 = 3 \text{ m} @ 30^\circ$$

$$\Delta \vec{s}_2 = 5 \text{ m} @ 60^\circ$$

if the second is made after the first, then the total displacement is just the sum of the two displacements. Now, the sum is not 8 m and it is not at 90° . What we have to do is to convert the two vectors to cartesian coordinates. They become

$$\Delta \vec{s}_1 = (3 \text{ m}) \cos(30^\circ) \cdot \hat{i} + (3 \text{ m}) \sin(30^\circ) \cdot \hat{j}$$

$$\Delta \vec{s}_2 = (5 \text{ m}) \cos(60^\circ) \cdot \hat{i} + (5 \text{ m}) \sin(60^\circ) \cdot \hat{j}$$

or

$$\Delta \vec{s}_1 = (2.5981 \text{ m}) \cdot \hat{i} + (1.5000 \text{ m}) \cdot \hat{j}$$

$$\Delta \vec{s}_2 = (2.5000 \text{ m}) \cdot \hat{i} + (4.3301 \text{ m}) \cdot \hat{j}$$

Their sum is this.

$$\Delta \vec{s}_{total} = (5.0981 \text{ m}) \cdot \hat{i} + (5.8301 \text{ m}) \cdot \hat{j}$$

In polar coordinates,

$$\Delta \vec{s}_{total} = \left(\sqrt{5.0981^2 + 5.8301^2} \text{ m} \right) @ \tan^{-1} \left(\frac{5.8301}{5.0981} \right)$$

or

$$\Delta \vec{s}_{total} = (7.7447 \text{ m}) @ 48.832^\circ$$

Geometrically,

