Note 1c Units

The fundamental units used in mechanics (study of force and motion) are these: kilogram (kg) for mass, meter (m) for length, and second (s) for time. Capitalization is important. All other units will be derived from these. Here are two simple derived units that we will use without a special name.

$$Velocity\text{: } \frac{m}{s}$$

$$Acceleration\text{: } \frac{m}{s^2}$$

Here are two, more complex, derived units that we will use with names.

$$Force\text{: } N(Newton) = \frac{kg \cdot m}{s^2}$$

$$Energy\text{: } J(Joule) = \frac{kg \cdot m^2}{s^2}$$

Here are some points about the units.

- Algebra with units is just like algebra with variables. You can add two numbers only if they have the same unit. For example, 1 meter plus 1 meter is 2 meters. However, 1 meter plus 1 second does not make sense.
- Another way in which units are useful is that they will tell you if you have made a mistake. The units on both sides of an equation must be the same. If they are not, there must be a mistake. A unit check at the end of a large number of algebraic steps help you find errors.
- Furthermore, you can also find the units of complex constants by knowing the units of everything else. For example, the universal gravitational constant G is in this equation has a unit.

  $$F = G\frac{m_1 m_2}{r^2}$$

  F represents force, the two m's are mass, and r is a distance, therefore, the unit of G must be the following.

  $$F = G\frac{m_1 m_2}{r^2} \quad \Rightarrow \quad G = F\frac{r^2}{m_1 m_2}$$

  $$unit\ of\ G = [G] = N\frac{m^2}{kg^2} = kg\frac{m}{s^2}\frac{m^2}{kg^2} = \frac{m^3}{kg \cdot s^2}$$

- The unit of the argument and the result of a function are unitless. For example, the position of a mass at the end of a spring looks like this.

  $$x(t) = A\sin(\omega t + \phi)$$

  The variable "x" is a position so its unit is meter. This means the unit of "A" must also meters as the sine function returns a unitless number. The argument of the sine function is an angle (which is not a unit but an indication of the way angle is measured whether it is measured in degrees or radians). Thus, "ωt" and "φ" are both unitless. Now, "t" is time in second so the unit of "ω" must be 1/second. The unit of "φ" is also unitless.

- To change units, multiply your value by 1 in the form of a unit conversion factor (ratio). The conversion factor is made up of the unit you want in the numerator and the unit you have in the denominator. For example, I have 3 days and I want to convert it seconds. I know how to go from days to hours, so let's do that first.

  $$3\ days\left(\frac{24\ hours}{1\ day}\right)$$

  Next, I know how to go from hours to seconds, so let do that next.

  $$3\ days\left(\frac{24\ hours}{1\ day}\right)\left(\frac{3600\ seconds}{1\ hour}\right)=2.592\times10^{5}\ s$$

  All of the conversion factors together then form the conversion from days to second.

  $$3\ days\left(\frac{8.64\times10^{4}\ seconds}{1\ day}\right)=2.592\times10^{5}\ s$$

Prefixes

Prefixes in from of a unit is used to reduces the number of zeroes required or to reduce the size of the exponent of a value. Here are the most used prefixes in this class.

micro (μ) = 10^{-6}

milli (m) = 10^{-3}

no prefix = 10^{0}

kilo (k) = 10^{+3}

mega (M) = 10^{+6}

Let's say we have a value of 1234.567 m. To 3 significant digits, this is 1230 m. This is also

$$1230\ m\left(\frac{1\ km}{1000\ m}\right)=1.23\ km$$

$$1.23\ km\left(\frac{1\ Mm}{1000\ km}\right)=0.00123\ Mm$$

This process is just reducing the size of the value by 1000 times and increasing the size of the unit by 1000 times. 1230 is reduced by 3 decimal places while the unit is increased by 3 decimal places. 1230 goes to 1.23 and "m" goes to "km".

To use a smaller unit, just do the opposite.

$$1230\ m\left(\frac{1000\ mm}{1\ m}\right)=1230000\ mm=1.23\times10^{6}\ mm$$

$$1.23\times10^{6}\ mm\left(\frac{1000\ \mu m}{1\ mm}\right)=1230000000\ \mu m=1.23\times10^{9}\ \mu m$$

Notice going in this direction, the number are harder to deal with. 1.23 km is much easier to handle than $1.23\text{x}10^{9}$ μm.