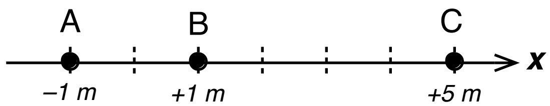
Note 2a Motion

Motion Variables

In order to describe how an object is moving, we need to measure the following variables. For example, let's look at an object that goes from point A to point B to point C on the x axis.



Position (vector)

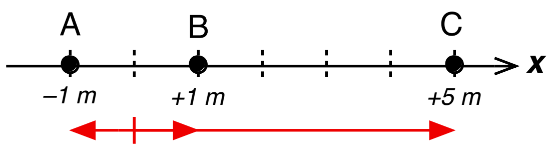
This is where the object is. We need this so that we can tract its change. At point A, the position is –1 m. At point B, the position is +1 m. At point C, the position is +5 m. In equation form, we could write them this way. Let s represent the position of the object.

$$\vec{s}_A = (-1\ m)\cdot\hat{i}$$

$$\vec{s}_B = (+1\ m)\cdot\hat{i}$$

$$\vec{s}_C = (+5\ m)\cdot\hat{i}$$

Here are the position vectors represented by arrows.



Displacement (vector)

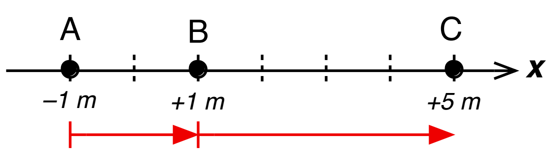
This is the change of the position. Two points. Changes are always represented by the symbol delta "Δ". The change is defined as the final value minus the initial value.

$$\Delta\vec{s} = \vec{s}_f - \vec{s}_i$$

In the above example, there are two displacements. The first is from A to B and the second is from B to C. The displacements are these.

$$\Delta\vec{s}_{AB} = \vec{s}_B - \vec{s}_A = (+1\ m)\cdot\hat{i} - (-1\ m)\cdot\hat{i} = (+2\ m)\cdot\hat{i}$$

$$\Delta\vec{s}_{BC} = \vec{s}_C - \vec{s}_B = (+5\ m)\cdot\hat{i} - (+1\ m)\cdot\hat{i} = (+4\ m)\cdot\hat{i}$$

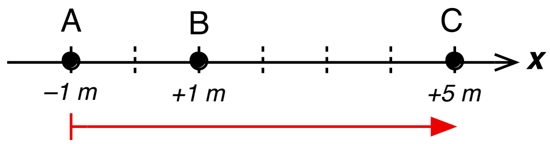


One can also look at the displacement from A to C if we don't care about position B.

$$\Delta\vec{s}_{AC} = \vec{s}_C - \vec{s}_A = (+5\ m)\cdot\hat{i} - (-1\ m)\cdot\hat{i} = (+6\ m)\cdot\hat{i}$$

This is also the total displacement from A to C.

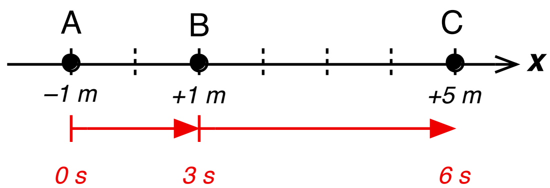
$$\Delta\vec{s}_{AC} = \Delta\vec{s}_{AB} + \Delta\vec{s}_{BC} = (+2\ m)\cdot\hat{i} + (+4\ m)\cdot\hat{i} = (+6\ m)\cdot\hat{i}$$



Time (scalar)

Moving from one position to another (or moving through a displacement) takes time.

Let's say that the time when the object is at point A is t_A = 0 s. The absolute time is arbitrary. When the object is at point B, t_B = 3 s. When the object is at point C, t_C = 6 s.

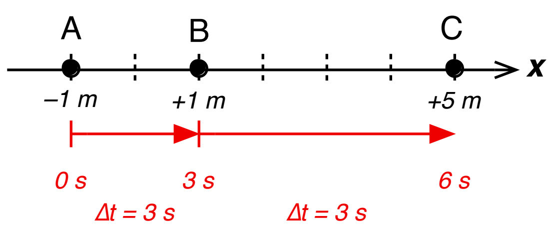


Elapsed Time (scalar)

All that physical processes care about is the elapsed time between the beginning and the end of the process. The elapsed time is

$$\Delta t = t_f - t_i$$

The displacement Δs_{AB} takes 3 seconds of elapsed time. The displacement Δs_{BC} also takes 3 seconds of elapsed time. The displacement Δs_{AC} takes 6 seconds of elapsed time.



Average Velocity (vector)

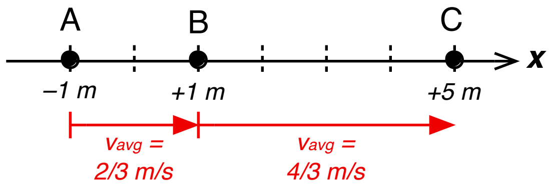
This is one of the variables we use to indicate that an object is in motion. If it has a non-zero average velocity, it is moving. This is the time rate at which the position of the object is changing. This is also the displacement per an elapsed time.

$$\vec{v}_{avg} = \frac{\Delta\vec{s}}{\Delta t}$$

This is an average because we don't measure exactly how the object moved from one point to the next. The larger the elapsed time, the more uncertain we are of the actual path of the object. In the example, the average velocities are these.

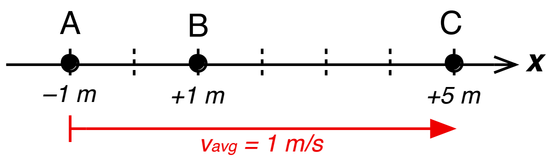
$$\vec{v}_{avg,AB} = \frac{\Delta\vec{s}_{AB}}{\Delta t} = \frac{(+2\ m)\cdot\hat{i}}{3\ s} = \left(\frac{2}{3}\ m/s\right)\cdot\hat{i}$$

$$\vec{v}_{avg,BC} = \frac{\Delta\vec{s}_{BC}}{\Delta t} = \frac{(+4\ m)\cdot\hat{i}}{3\ s} = \left(\frac{4}{3}\ m/s\right)\cdot\hat{i}$$



The average velocity for the entire trip is

$$\vec{v}_{avg,AC} = \frac{\Delta\vec{s}_{AC}}{\Delta t} = \frac{(+6\ m)\cdot\hat{i}}{6\ s} = (1\ m/s)\cdot\hat{i}$$



Average Acceleration (vector)

This is the rate at which the velocity of the object is changing. Notice in the above diagram, the average velocity point in different directions at different times even though the lengths of the actors are the same. Thus, there is a change.

To measure the average acceleration, we have to measure the actual velocity at the two specific times. We don't have that from just the positions.

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

The Velocity (vector)

In order to get the actual **velocity**, we have to shrink the elapsed time go to zero. This will produce an average velocity over one specific time. This is also known as the **instantaneous velocity**.

$$\vec{v} = \lim_{\Delta t \to 0} \vec{v}_{avg} = \lim_{\Delta t \to 0} \frac{\Delta \vec{s}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{s}\left(t_f\right) - \vec{s}\left(t_i\right)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{s}\left(t_i + \Delta t\right) - \vec{s}\left(t_i\right)}{\Delta t} = \frac{d\vec{s}}{dt}$$

There is one special case when the average velocity and the instantaneous velocity are the same. This happens when the velocity is constant.

The Acceleration (vector)

In order to get the actual acceleration at an exact time, we again shrink the elapsed time to zero. This will produce an average acceleration over one specific time. This is also known as the **instantaneous acceleration**.

$$\vec{a} = \lim_{\Delta t \to 0} \vec{a}_{avg} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{v}\left(t_f\right) - \vec{v}\left(t_i\right)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{v}\left(t_i + \Delta t\right) - \vec{v}\left(t_i\right)}{\Delta t} = \frac{d\vec{v}}{dt}$$

The average acceleration and the instantaneous acceleration are the same again when the acceleration is constant.

Summary

Motion is defined in general by these two definitions for continuous motion functions.

$$\vec{v} = \frac{d\vec{s}\left(t\right)}{dt}$$

$$\vec{a} = \frac{d\vec{v}\left(t\right)}{dt}$$

In real life, we have to measure displacements and elapsed times, so we have to use averages instead.

$$\vec{v}_{avg} = \frac{\Delta \vec{s}}{\Delta t}$$

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$