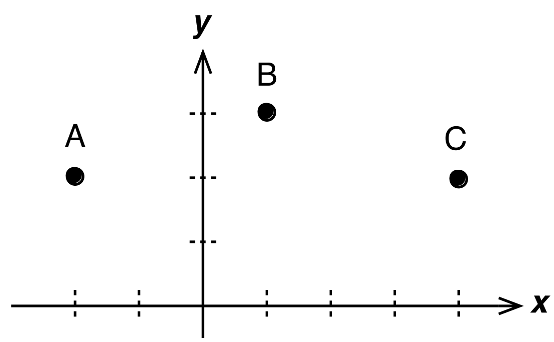
Note 2d Motion in 2D

Motion Variables

In order to describe how an object is moving, we need to measure the following variables. For example, let's look at an object that goes from point A to point B to point C.



Position (vector)

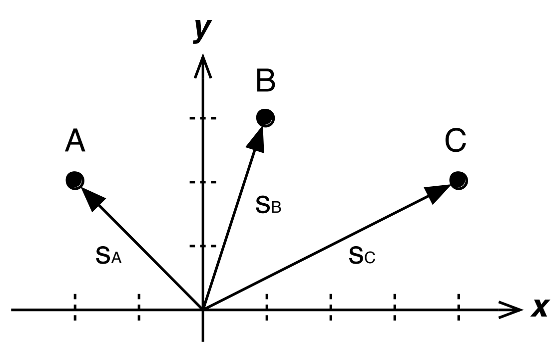
This is where the object is. We need this so that we can tract its change. At point A, the position is ⟨–2 m, 2 m⟩. At point B, the position is ⟨1 m, 3 m⟩. At point C, the position is ⟨4 m, 2 m⟩. In equation form, we could write them this way. Let s represent the position of the object.

$$\vec{s}_A = \langle -2\ m,\ 2\ m \rangle = (-2\ m)\cdot\hat{i} + (2\ m)\cdot\hat{j}$$

$$\vec{s}_B = \langle 1\ m,\ 3\ m \rangle = (1\ m)\cdot\hat{i} + (3\ m)\cdot\hat{j}$$

$$\vec{s}_C = \langle 4\ m,\ 2\ m \rangle = (4\ m)\cdot\hat{i} + (2\ m)\cdot\hat{j}$$

Here are the position vectors represented by arrows.



Displacement (vector)

This is the change of the position. Two points. Changes are always represented by the symbol delta "Δ". The change is the final value minus the initial value. Thus, the displacement, as defined by the change in the position, is this.

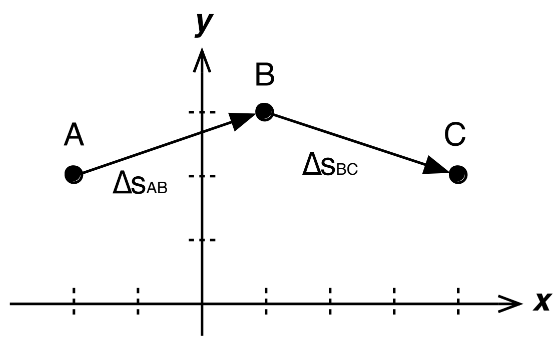
$$\Delta\vec{s} = \vec{s}_f - \vec{s}_i$$

In the above example, there are two displacements. The first is from A to B and the second is from B to C. The displacements are these.

$$\Delta\vec{s}_{AB} = \vec{s}_B - \vec{s}_A = (1\ m)\cdot\hat{i} + (3\ m)\cdot\hat{j} - \left[(-2\ m)\cdot\hat{i} + (2\ m)\cdot\hat{j}\right] = (3\ m)\cdot\hat{i} + (1\ m)\cdot\hat{j}$$

$$\Delta\vec{s}_{BC} = \vec{s}_C - \vec{s}_B = (4\ m)\cdot\hat{i} + (2\ m)\cdot\hat{j} - \left[(1\ m)\cdot\hat{i} + (3\ m)\cdot\hat{j}\right] = (3\ m)\cdot\hat{i} + (-1\ m)\cdot\hat{j}$$

Here are the displacement vectors.

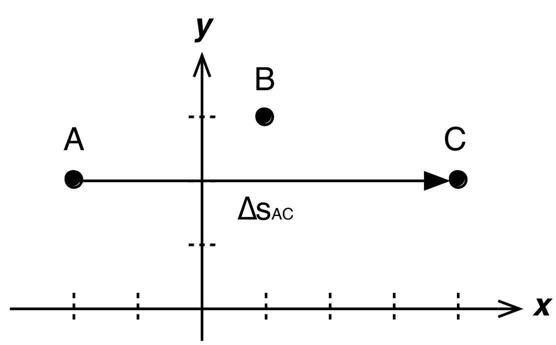


One can also look at the displacement from A to C if we don't care about position B.

$$\Delta\vec{s}_{AC} = \vec{s}_C - \vec{s}_A = (4\ m)\cdot\hat{i} + (2\ m)\cdot\hat{j} - \left[(-2\ m)\cdot\hat{i} + (2\ m)\cdot\hat{j}\right] = (6\ m)\cdot\hat{i} + (0\ m)\cdot\hat{j}$$

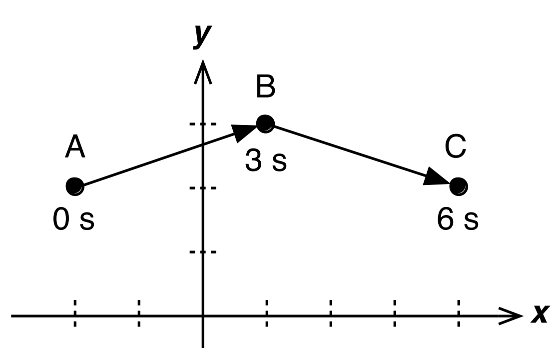
This is also the total displacement from A to C.

$$\Delta\vec{s}_{AC} = \Delta\vec{s}_{AB} + \Delta\vec{s}_{BC} = (3\ m)\cdot\hat{i} + (1\ m)\cdot\hat{j} + (3\ m)\cdot\hat{i} + (-1\ m)\cdot\hat{j} = (6\ m)\cdot\hat{i} + (0\ m)\cdot\hat{j}$$



Time (scalar)

Moving from one position to another (or moving through a displacement) takes time. Let's say that the time when the object is at point A is $t_A = 0$ s. The absolute time is arbitrary. When the object is at point B, $t_B = 3$ s. When the object is at point C, $t_C = 6$ s.



Elapsed Time (scalar)

This is the the change in the time. For all physical processes, this is what matters. Again, this is a difference.

$$\Delta t = t_f - t_i$$

The displacement Δs_{AB} takes 3 seconds of elapsed time. The displacement Δs_{BC} also takes 3 seconds of elapsed time. The displacement Δs_{AC} takes 6 seconds of elapsed time.

Average Velocity (vector)

This is one of the variables we use to indicate that an object is in motion. If it has a non-zero average velocity, it is moving. This is the time rate at which the position of the object is changing. This is also the displacement per an elapsed time.

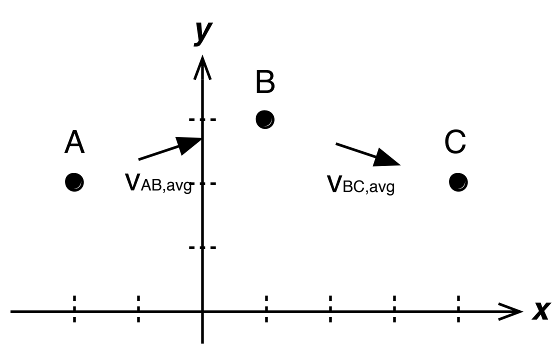
$$\vec{v}_{avg} = \frac{\Delta \vec{s}}{\Delta t}$$

This is an average because we don't measure exactly how the object moved from one point to the next. The larger the elapsed time, the more uncertain we are of the actual path of the object. In the example, the average velocities are these.

$$\vec{v}_{avg,AB} = \frac{\Delta \vec{s}_{AB}}{\Delta t} = \frac{(3\ m)\cdot\hat{i} + (1\ m)\cdot\hat{j}}{3\ s} = (1\ m/s)\cdot\hat{i} + (1/3\ m/s)\cdot\hat{j}$$

$$\vec{v}_{avg,BC} = \frac{\Delta \vec{s}_{BC}}{\Delta t} = \frac{(3\ m)\cdot\hat{i} + (-1\ m)\cdot\hat{j}}{3\ s} = (1\ m/s)\cdot\hat{i} + (-1/3\ m/s)\cdot\hat{j}$$

Here are the average velocity vectors represented by arrows. They do point in the directions of their displacements.



Average Acceleration (vector)

This is the rate at which the velocity of the object is changing. Notice in the above diagram, the average velocity point in different directions at different times even though the lengths of the actors are the same. Thus, there is a change.

To measure the average acceleration, we have to measure the actual velocity at the two specific times. We don't have that from just the positions.

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

In general, the average acceleration points in the direction of the change in the velocity.

$$\Delta\vec{v} = \left(1\ m/s\right)\cdot\hat{i} + \left(-1/3\ m/s\right)\cdot\hat{j} - \left[\left(1\ m/s\right)\cdot\hat{i} + \left(1/3\ m/s\right)\cdot\hat{j}\right] = \left(0\ m/s\right)\cdot\hat{i} - \left(2\,/\,3\ m/s\right)\cdot\hat{j}$$